

Asset Allocation and Risk Management in a Bimodal World

For many years, market participants have confidently relied on a modeling framework that considered a single “equilibrium” – or system in which competing influences are in balance – when constructing portfolios. The most popular assumption under such an equilibrium model has been a probability distribution curve that is unimodal (i.e. has only one peak) and a mean that coincides with this single peak.

However, a key impact of the recent bout of crises hitting global markets has been the possibility of the emergence of multiple equilibria, which might happen if one or another competing force takes the upper hand.

For example, the policy risk that pervades the markets today causes high correlations among asset classes and a temperament of “risk on/risk off” among investors. This phenomenon can be traced to the connectedness of markets, the ease by which market participants can access these connected markets, and the speed of assimilation of information in response to political events. (See V. Bhansali, *The Ps of Pricing and Risk Management, Revisited*, Journal of Portfolio Management, Vol. 36, No. 2, Winter 2010.) This environment creates the possibility of multiple equilibria in the market, as well as trends that move markets between these equilibria, and once settled, restraining forces that trap markets in those equilibria (See V. Bhansali, *Market Crises -- Can the Physics of Phase Transitions and Symmetry Breaking Tell Us Anything Useful?*, Journal of Investment Management, 2009).

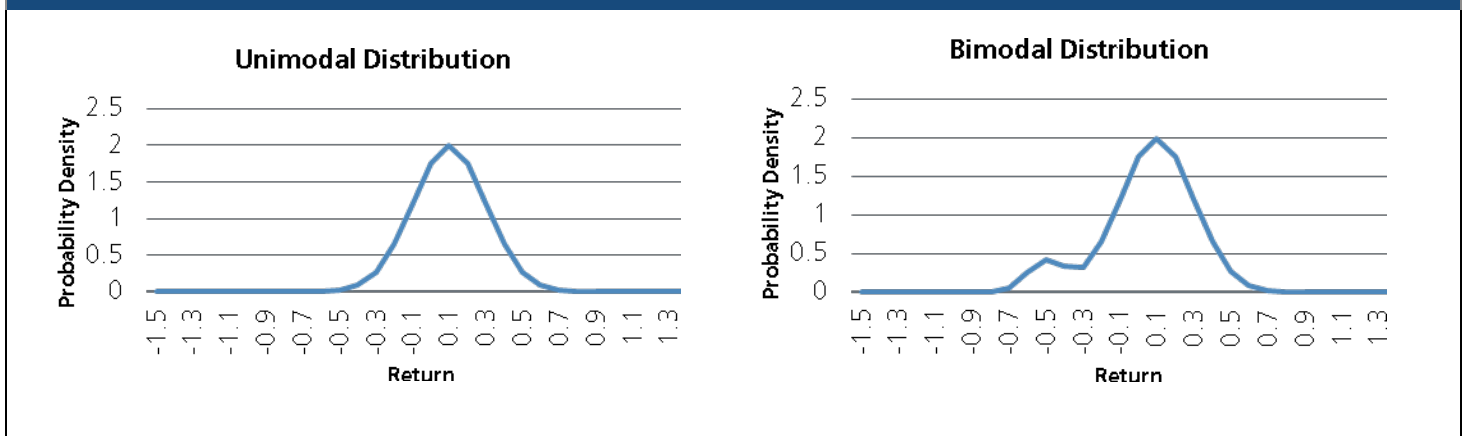
Even though predicting which force will win is next to impossible given the real-time evolution of the interaction between markets and policy, we can still ask an important question: What would happen if the distribution of returns from a hypothetical portfolio looked more like the one shown in the chart on the right of Figure 1, i.e. a “bimodal” distribution with more than one peak? The bimodal distribution has two peaks, and interestingly, even though it is generated as the result of mixing two normal distributions, each from a different regime, it can exhibit both fat tails (a higher probability of larger losses due to unusual events results in a “fat tail” on the left side of the distribution curve) and skewness (a lack of symmetry between the left and right sides of the peak).



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Figure 1: Normal and Bimodal Distribution of Returns



Hypothetical examples for illustrative purposes only.
Source: PIMCO

Unimodal versus Bimodal Distributions

When constructing the “normal” returns chart we used the long-term history of the S&P 500 Index as a proxy to approximate the stock market (1951 through 2010) and assumed a normal distribution: 10% average annual return and 20% volatility, as measured by standard deviation. For the bimodal distribution, we assumed that there were two regimes: the first is the one shown in our normal distribution (10% average return and 20% volatility), but the second “bad” regime is one where equities go down 50%, and then become trapped in that new scenario. A group of Deutsche Bank analysts led by Vinay Pande has been writing for a few years that equity market returns realized in the recent past are bimodal. Indeed, in one of our client meetings, my colleague Marc Seidner raised the possibility that the future looks a lot more bimodal than the past ever did, based on this evidence and similar data.

To illustrate this, we assumed for our example that there was only a 10% chance of the second regime happening, but once it happens the environment is a sticky, local equilibrium – a “hole that is hard to climb out of.” The interested reader can make up an infinite number of plausible scenarios such as these, and is encouraged to question accepted lore of asset allocation and portfolio construction under such multimodal distributions. In this note we will attempt to do exactly such an exercise.

For the bimodal distribution that results from combining the normal and bad regimes, the average return is 4% and the volatility is 26% (versus a 10% average return and 20% volatility for the unimodal normal distribution). This is simply because the bad regime has sufficient weight to reduce the overall returns. There is also negative skewness (of -0.58) in the bimodal curve versus zero skewness for the normal distribution, and excess kurtosis (a measure of whether data are peaked or flat) of 0.19 over the normal

distribution, reflecting the magnitude of unlikely outcomes, or how fat the tails are (under “old normal” circumstances they are rather flat). All of these statistics are not too far from what one would glean from looking at the implied distributions from current option prices in broad equity indices; but with the important difference that traditional option pricing models get their fat tails and skewness from building in the skew ex-post on top of a unimodal distribution.

None of these observations should seem surprising if one realizes that a mixture of two normal regimes can yield a result that is abnormal. Fat tails and negative skewness can arise from even the mere *possibility* of multiple equilibria, even though both equilibria in themselves are normal. This practice of generating very complex distributions from mixtures of simple, normal distributions is well known among statisticians and has applications in many fields of practical import: medicine, astronomy and casino gambling to name a few. In the present context, the two “normals” are the mixture of the old normal and the New Normal that PIMCO has been talking about for a few years now.

Implications for Portfolio Construction and Hedging

How does this bimodality apply to portfolio construction in the world of high volatility and multiple equilibria that we see today? Here is what we find when we apply it to two problems of finance: optimal asset allocation and option pricing.

Optimal Allocation to Risky Assets: If we start with an assumption that we would allocate 50% of the portfolio to equities in the unimodal case, what would the optimal allocation be in the bimodal case, assuming our risk preferences are unchanged? By following a very traditional portfolio optimization exercise which involves a little bit of math, the answer turns out to be that the optimal

allocation would be only 10%! In other words, one would have to de-risk by almost 80% from the current optimal allocation to arrive at the mathematically optimal result (see disclosures at the end of this article for a more detailed explanation of our computations). The prospect of being trapped in a low return, low probability event requires us to, as Mohamed El-Erian would say, “generally play defense and selectively play offense.”

Pricing of Options on Tails: If we started with an assumption of unimodality and the real distribution turned out to be the bimodal one, how mispriced would put options on the tails be in retrospect? Our research shows that a typical unimodal distribution just cannot be tweaked large enough to make it come out with the price of a put option one would likely get if the real world turned out to be bimodal. A portfolio manager pricing such tail options armed with traditional unimodal distributions would wrongly think that the tail options were “expensive” (tail options will generally tend to be underpriced when based on a unimodal distribution but significantly higher when derived from bimodal distributions).

(Again, for the mathematically inclined we priced the options by mathematically summing the put payoff over all the probability-weighted outcomes from the two distributions.) These two illustrations show that the very possibility of a bimodal outcome forces you to de-risk directly, i.e. by reducing the allocation to risky assets; or to build in tail hedging, which might look expensive by traditional measures, but may turn out to actually be “cheap” in a bimodal world.

For all these reasons, we believe that the core building blocks of asset allocation and option pricing should incorporate the possibility of multimodality. To follow a traditional approach in a world that is so exposed to the possibility of multiple equilibria is to ignore the reality of today’s markets.

Optimal Allocations: *For those who are mathematically inclined, the computation of the optimal allocation to risky assets begins by specifying a very standard power utility function, $W^{(1-a)/(1-a)}$, which represents the preferences of a typical investor with constant relative risk aversion and W is the initial wealth of the investor. We used a typical parameter of $a=5$. The investor has a choice to allocate between the risky asset (stocks) and keep money in cash at a return rate of 25 basis points annually. (The curious reader is encouraged to experiment with other assumptions).*

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